Self-trapping of strong electromagnetic beams in relativistic plasmas

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Interaction of an intense electromagnetic (em) beam with a relativistic electron-positron (e-p) plasma is investigated. It is shown that the thermal pressure brings about a fundamental change in the dynamics—localized, high amplitude, em field structures, not accessible to a cold (but relativistic) plasma, can now be formed under well-defined conditions. The possibilities of trapping em beams in self-guiding regimes to form stable two-dimensional solitonic structures in a pure e-p plasma are worked out.

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Relativistic electron-positron (e-p) dominated plasmas may be created in a variety of astrophysical situations. Electron-positron pair production cascades are believed to occur in pulsar magnetospheres [1]. The e-p plasmas are also likely to be found in the bipolar outflows (jets) in active galactic nuclei (AGN) [2], and at the center of our own galaxy [3]. The presence of e-p plasma is also argued in the MeV epoch of the early universe. In the standard cosmological model, temperatures in the MeV range ($T \sim 10^{10}$ K–1 MeV) prevail up to times $t=1$ s after the big bang [4].

Studying wave self-modulation and soliton formation in e-p plasmas is, therefore, of considerable importance in understanding the overall dynamics of these systems. The existence of stable localized envelope solitons of em radiation has been suggested as a potential mechanism for the production of micropulses in AGN and pulsars [5]. Localized solitons created in the plasma dominated era are also invoked to explain the observed inhomogeneities of the visible universe [6,7].

In Refs. [6,8] it was argued that localized solitons can be formed if the interaction between the em field and acoustic phonons is taken into account—envelope solitons propagating with subsonic velocities may then emerge. However, it is conceivable that soliton solutions obtained in a one-dimensional (1D) formulation will turn out to be unstable in higher dimensions.

It is, therefore, a matter of some priority that we explore the possibility of finding stable multidimensional soliton solutions in e-p plasmas. A 3D dynamics of envelope solitons of arbitrary strong em fields in such a plasma contaminated by a small fraction of heavy ions was analyzed in Ref. [7]. It was shown that, in a transparent plasma, em pulses with $L_\parallel \ll L_\perp$ (where $L_\parallel$ and $L_\perp$ are, respectively, the characteristic longitudinal and transverse spatial dimensions of the field) may propagate as stable, nondiffracting and nondispersing, objects (light bullets). These bullets are exceptionally robust: they can emerge from a large variety of initial field distributions, and are remarkably stable. In these studies, the em field is pulse like with a longitudinal localization much stronger than the transverse; the ponderomotive pressure force of such a pulse causes the plasma to move in the direction of propagation, leading to large density bunching. The localization is brought about by the charge separation electric field (usually absent in a pure e-p plasma) created by the ionic contamination—the thermal pressure is not able to do this because the bunches move with a speed close to the speed of light.

In the present paper we investigate the dynamics of em fields in the opposite limit i.e., we study the "beam" ($L_\perp \ll L_\parallel$) rather than the pulse ($L_\perp \gg L_\parallel$) dynamics. For a plasma transparent to the beam, we apply a fully relativistic hydrodynamic model to demonstrate the possibility of beam self-trapping leading to the formation of stable 2D solitonic structures in a pure e-p plasma. The high-frequency pressure force of the em field (tending to completely expel the pairs radially from the region of localization) is now overwhelmed by the thermal pressure force, which opposes the radial expansion of the plasma, creating conditions for the formation of the stationary self-guiding regime of beam propagation.

If the velocity distribution of the particles of species α ($=e,p$, where e,p denote, respectively, electrons and positrons) is taken to be a local relativistic Maxwellian, the hydrodynamics of such fluids is described by

$$\frac{\partial}{\partial t}(G_\alpha \boldsymbol{p}_\alpha) + m_{0\alpha}c^2 \boldsymbol{\nabla}(G_\alpha \gamma_\alpha) = e_\alpha \boldsymbol{E} + [\boldsymbol{u}_\alpha \times \boldsymbol{\Omega}_\alpha] \tag{1}$$

and

$$\frac{\partial \boldsymbol{\Omega}_\alpha}{\partial t} = \boldsymbol{\nabla} \times [\boldsymbol{u}_\alpha \times \boldsymbol{\Omega}_\alpha] \tag{2}$$

(see [7,9] for the details), where $\boldsymbol{\Omega}_\alpha = (e_\alpha/c)\boldsymbol{B} + \boldsymbol{\nabla} \times (G_\alpha \boldsymbol{p}_\alpha)$ is the so called generalized vorticity. Here $\boldsymbol{p}_\alpha = \gamma_\alpha m_{0\alpha} \boldsymbol{u}_\alpha$ is the hydrodynamical momentum, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields, $G_\alpha = K_3(z_\alpha)/K_2(z_\alpha)$ is the "effective mass," K_2 and K_3 are, respectively, the modified Bessel functions of the second and third order, and $z_\alpha = m_{0\alpha}c^2/T_\alpha$; $m_{0\alpha}$ and T_α are the particle invariant rest mass and temperature, respectively, and n_α is the density in the laboratory frame of the fluid of species α. The hydrodynamical velocity $\boldsymbol{u}_\alpha$ and the relativistic factor γ_α are related to the momentum by the standard relations $\boldsymbol{u}_\alpha = \boldsymbol{p}_\alpha / m_{0\alpha}\gamma_\alpha$, $\gamma_\alpha = (1 + \boldsymbol{p}_\alpha^2/m_{0\alpha}^2 c^2)^{1/2}$.

Notice that in Eq. (1) the thermal pressure P_α $(=n_\alpha T_\alpha/\gamma_\alpha)$ appears through the temperature dependent factor G defined by $\gamma_\alpha \nabla P_\alpha = m_{0\alpha}c^2 n_\alpha \nabla G_\alpha$. The system of Eqs. (1), (2) is augmented by the equation of state

$$\frac{n_\alpha}{\gamma_\alpha}\frac{z_\alpha}{K_2(z_\alpha)}\exp(-G_\alpha z_\alpha)=\text{const.} \tag{3}$$

For the current effort, we apply Eqs. (1)–(3) for wave processes in an unmagnetized plasma. From Eq. (2) it follows that if the generalized vorticity is initially zero ($\mathbf{\Omega}_\alpha = \mathbf{0}$) everywhere in space, it remains zero for all subsequent times. We assume that before the em radiation is ‘‘switched on’’ the generalized vorticity of the system is zero.

In what follows, for notational convenience, we replace the subscripts (e) and (p) by superscripts $(-)$ and $(+)$. Also, it is convenient to introduce the temperature dependent momentum $\mathbf{\Pi}_\alpha = G_\alpha \boldsymbol{p}_\alpha$ and relativistic factor $\Gamma_\alpha = G_\alpha \gamma_\alpha$. We assume that the equilibrium state of the plasma is characterized by an overall charge neutrality $n_\infty^- = n_\infty^+ = n_\infty$, where n_∞^- and n_∞^+ are the unperturbed number densities of the electrons and positrons in the far region of the em beam localization. In most mechanisms for creating e-p plasmas, the pairs appear simultaneously and due to the symmetry of the problem it is natural to assume that $T_\infty^- = T_\infty^+ = T_\infty$, where T_∞^- and T_∞^+ are the respective equilibrium temperatures.

We shall assume that for the radiation field of interest the plasma is underdense and transparent, i.e., $\epsilon = \omega_e/\omega \ll 1$, where ω is the mean frequency of em radiation and $\omega_e = (4\pi e^2 n_\infty/m_{0e})^{1/2}$ is the plasma frequency. We will display the entire set in terms of potentials (the Coulomb gauge $\nabla\cdot\boldsymbol{A}=0$ will be used) $\boldsymbol{E} = -c^{-1}\partial_t \boldsymbol{A} - \nabla\phi, \boldsymbol{B} = \nabla\times\boldsymbol{A}$, and the dimensionless quantities $\tilde{t} = \omega t$, $\tilde{\boldsymbol{r}} = (\omega/c)\boldsymbol{r}$, $\tilde{T}^\pm = T^\pm/m_{0e}c^2$, $\tilde{\boldsymbol{A}} = e\boldsymbol{A}/(m_{0e}c^2)$, $\tilde{\phi} = e\phi/m_{0e}c^2$, $\tilde{\mathbf{\Pi}}^\pm = \mathbf{\Pi}^\pm/(m_{0e}c)$, and $\tilde{n}^\pm = n^\pm/n_\infty$. Suppressing the tilde, we arrive at the dimensionless equations

$$\frac{\partial \mathbf{\Pi}^\pm}{\partial t} + \nabla\Gamma^\pm = \mp\frac{\partial \boldsymbol{A}}{\partial t} \mp \nabla\phi, \tag{4}$$

$$\frac{\partial^2 \boldsymbol{A}}{\partial t^2} - \Delta\boldsymbol{A} + \frac{\partial}{\partial t}\nabla\phi - \epsilon^2(\boldsymbol{J}^+ - \boldsymbol{J}^-) = 0, \tag{5}$$

$$\Delta\phi = \epsilon^2(n^- - n^+), \tag{6}$$

$$\frac{\partial n^\pm}{\partial t} + \nabla\cdot\boldsymbol{J}^\pm = 0 \tag{7}$$

with $\boldsymbol{J}^\pm = n^\pm \mathbf{\Pi}^\pm/\Gamma^\pm$ and $\Gamma^\pm = \sqrt{(G^\pm)^2 + (\mathbf{\Pi}^\pm)^2}$. The species equation of state is

$$\frac{n^\pm}{\Gamma^\pm f(T^\pm)} = \frac{1}{\Gamma_\infty^\pm f(T_\infty)}, \tag{8}$$

where $f(T^\pm) = [K_2(1/T^\pm)T^\pm/G^\pm]\exp[G^\pm/T^\pm]$.

Of various techniques that could be invoked to investigate Eqs. (4)–(8) to study the self-trapping of high-frequency em radiation propagating along the z axis, we choose the method presented in the excellent paper by Sun *et al.* [10]. The method is based on the multiple scale expansion of the equations in the small parameter ϵ. Assuming that all variations are slow compared to the variation in ‘‘moving’’ with a velocity a, variable $\xi = z - at$, we expand all quantities $Q = (\boldsymbol{A}, \phi, \mathbf{\Pi}^\pm, n^\pm, \dots)$ as $Q = Q_0(\xi, x_1, y_1, z_2) + \epsilon Q_1(\xi, x_1, y_1, z_2)$, where $(x_1, y_1, z_2) = (\epsilon x, \epsilon y, \epsilon^2 z)$ denote the directions of slow change, and $a_1 = (a^2-1)/\epsilon^2 \sim 1$. We further assume that the high-frequency em field is circularly polarized, $\boldsymbol{A}_{0\perp} = \frac{1}{2}(\hat{\boldsymbol{x}} + i\hat{\boldsymbol{y}})A\exp(i\xi/a) + \text{c.c.}$ Here A is the slowly varying envelope of the em beam, $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ denote unit vectors, and c.c. is the complex conjugate. We now give a short summary of the steps in the standard multiple-scale methodology (Ref. [10]). To lowest order in ϵ, we obtain the following. The transverse (to the direction of em wave propagation z) component of Eq. (4) reduces to $\mathbf{\Pi}_{0\perp}^\pm = \mp\boldsymbol{A}_{0\perp}$ and for the longitudinal components we get

$$-a\frac{\partial \Pi_{0z}^\pm}{\partial\xi} + \frac{\partial\Gamma_0^\pm}{\partial\xi} = \mp(-a)\frac{\partial A_{0z}}{\partial\xi} \mp \frac{\partial\phi_0}{\partial\xi}. \tag{9}$$

Equations (5) and (6) yield $\partial_\xi \nabla_\perp \phi_0 = \partial_\xi^2 \phi_0 = \partial_\xi A_{0z} = 0$, where $\nabla_\perp$ is the perpendicular Laplacian in (x_1, y_1). These relations imply that ϕ and A_{0z} do not depend on the fast variable ξ. For the self-trapping problem, we can assume that $A_{0z} = \Pi_{0z}^\pm = 0$ [10]. From Eq. (9), and from the lowest-order continuity Eq. (7), we obtain $\partial_\xi \Gamma_0^\pm = \partial_\xi n_0^\pm = 0$, i.e., $\Gamma_0^\pm$ and $n_0^\pm$ also do not depend on the fast variable ξ.

To the next order (in ϵ), the transverse component of Eq. (4) reads

$$-a\frac{\partial \mathbf{\Pi}_{1\perp}^\pm}{\partial\xi} + \nabla_\perp\Gamma_0^\pm = \mp(-a)\frac{\partial \boldsymbol{A}_{1\perp}}{\partial\xi} \mp \nabla_\perp\phi_0. \tag{10}$$

Averaging over the fast variable ξ we obtain $\nabla_\perp\Gamma_0^\pm = \mp\nabla_\perp\phi_0$, yielding the trivial solution $\phi_0 = 0$ and $\Gamma_0^\pm = \Gamma_0 = \text{const}$. Note that from Eqs. (6) and (8), we can deduce that $n_0^+ = n_0^- = n_0$ and $T_0^+ = T_0^- = T_0$.

Thus, as one would expect, the low-frequency motion of the e-p plasma is driven by the ponderomotive pressure ($\sim\mathbf{\Pi}_{0\perp}^2$) of the high-frequency em field and this force, being the same for the electrons and positrons, does not cause charge separation. It is also evident that, because of the symmetry between the electron and positron fluids, their temperatures, being initially equal, will remain equal during the evolution of the system. The relation between the em field and the temperature can be found using the equation $\Gamma_0 = \text{const}$ obtained above. By choosing the constant by requiring that at infinity $A \to 0$ and $T_0 \to T_\infty$, we derive

$$G^2(T_0) = G^2(T_\infty) - |A|^2. \tag{11}$$

Note that the condition $G^2(T_\infty) > |A|^2$ prevents wave breaking from occurring.

We are now ready to deal with the equation for the slowly varying envelope $\boldsymbol{A}$ of the em beam. To the lowest order in ϵ, one finds from Eq. (5)

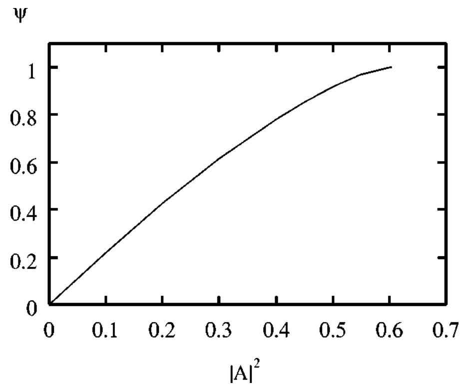

FIG. 1. The nonlinearity function Ψ versus $|A|^2$ for $T_\infty = 0.1$.

$$a_1 \frac{\partial^2 \boldsymbol{A}_{0\perp}}{\partial \xi^2} - \boldsymbol{\nabla}_\perp^2 \boldsymbol{A}_{0\perp} - 2\frac{\partial^2 \boldsymbol{A}_{0\perp}}{\partial \xi \partial z_2} + 2\frac{n_0(T_0)}{G_\infty}\boldsymbol{A}_{0\perp} = \boldsymbol{0}. \quad (12)$$

For deriving this equation, we used the relation $\Gamma_0 = \sqrt{G^2(T_0)+|A|^2} = G_\infty$ and $n_0(T_0) = f(T_0)/f(T_\infty)$ which follows from Eq. (8). In terms of the slowly varying envelope A, Eq. (12) can be reduced to the following equation:

$$i\frac{\partial A}{\partial z} + \boldsymbol{\nabla}_\perp^2 A + \Psi A = 0, \quad (13)$$

where $\Psi = 1 - n_0(T_0)$ represents the generalized nonlinearity. Here the subscripts for the variables (z_2, x_1, y_1) are dropped for simplicity and renormalization of the variables $z \to z G_\infty$, $r_\perp \to r_\perp \sqrt{G_\infty/2}$ is made. We also assumed without loss of generality that $(a^2-1)/\epsilon^2 a^2 = 2/G_\infty$, which in dimensional units coincides with the linear dispersion relation of the em wave in an *e-p* plasma, namely, $\omega^2 = 2\omega_e^2/G_\infty + k^2c^2$ provided that $a = \omega/kc$.

Thus, the dynamics of em beams in hot relativistic *e-p* plasma has become accessible within the context of a generalized nonlinear Schrödinger equation (NSE) (13). We seek the localized 2D soliton solutions of Eq. (13), and analyze the stability of such solutions. The companion equation (11) can be viewed as a transcendental algebraic relation between T_0 and $|A|^2$. Thus we conclude that Ψ is a function of $|A|^2$ [$\Psi = \Psi(|A|^2)$]. Unfortunately, it is not possible, in general, to derive an explicit analytic relation $\Psi = \Psi(|A|^2)$ for arbitrary value of T_∞. Some qualitative deductions readily follow. Equation (11) shows that the presence of em radiation reduces the temperature T_0. Since $df(T_0)/dT_0 > 0$, we conclude that the plasma density $n_0(T_0)$ is also reduced in the region of em field localization which is in accordance with adiabatic motion of the plasma. For higher strength of the em field, a complete expulsion of plasma, i.e., plasma cavitation, can take place $(n_0 \to 0)$; this was predicted in Ref. [11]. Thus the nonlinearity function Ψ shows a saturating character with increase of the em field strength [note that the present model is valid provided $|A|^2/(G_\infty^2 - 1) \leqslant 1$]. To illustrate, we exhibit in Fig. 1 a plot of Ψ versus $|A|^2$ for $T_\infty = 0.1$. One can see that the nonlinearity function indeed saturates at high intensity. For small temperatures, we can even obtain an analytic expression for the function Ψ. Remembering $T_0 \leqslant T_\infty$, assuming $T_\infty \ll 1$, and using $n_0(T_0) = f(T_0)/f(T_\infty)$ along with the asymptotic expansions $G_0(\approx 1 + 5T_0/2)$ and f $(\approx T_0^3)$, we derive for the nonlinearity function

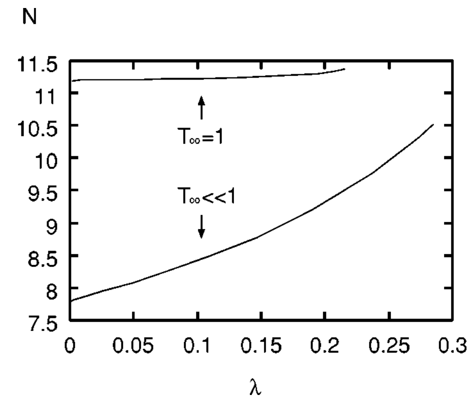

FIG. 2. The beam power N versus λ for $T_\infty \ll 1$ and $T_\infty = 1$.

$$\Psi = 1 - \left(1 - \frac{|A|^2}{5T_\infty}\right)^{3/2}. \quad (14)$$

Equations (13) and (14) admit a stationary, nondiffracting axially symmetric solution of the form $A/\sqrt{5T_\infty} = U(r)\exp(i\lambda z)$ where $r = (x^2+y^2)^{1/2}$ and λ is the nonlinear wave-vector shift. The radially dependent envelope $U(r)$ obeys an ordinary nonlinear differential equation

$$\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} - \lambda U + \Psi(U^2)U = 0, \quad (15)$$

where $\Psi = 1 - (1-U^2)^{3/2}$. This equation corresponds to a boundary value problem with the following boundary conditions: U has its maximum U_m at $r=0$, and $U \to 0$ as $r \to \infty$. We remind the reader that it was shown in a seminal paper of Vakhitov and Kolokolov [12] that such solutions exist for arbitrary saturating nonlinearity functions Ψ, provided that the eigenvalue λ satisfies $0 < \lambda < \Psi_m$, where Ψ_m is the maximal value of the nonlinearity function. We consider only the lowest-order nodeless solution of Eq. (15), i.e., the "ground state," which is positive and monotonically decreasing with increasing r. In the asymptotic region the solution must decay as $U_{r\to\infty} \sim \exp(-\sqrt{\lambda} r)/\sqrt{\lambda r}$. Our nonlinearity function Ψ has a maximum $\Psi_m = 1$ found at U_m $(=1)$, i.e., at the maximally allowed strength of the field. As a consequence the upper bound of the propagation constant λ_c must satisfy $\lambda_c < \Psi_m$. Numerical simulations show that the amplitude of the ground state solution $U_m = U(r=0,\lambda)$ is a growing function of λ and it acquires its maximum value $(=1)$ at $\lambda = \lambda_c \approx 0.29$. The solution represents a trapped, localized em solitary beam. The stability of the solitonic solutions can be investigated using the criterion of Vakhitov and Kolokolov [12]—the soliton is stable against small, arbitrary perturbations if $dN/d\lambda > 0$, where N is the power of the trapped mode: $N(\lambda) = 2\pi\int_0^\infty dr\, r U^2(r,\lambda)$. In Fig. 2 we plot the numerically obtained solutions of N for various λ. Since the curve has positive slope everywhere, the corre-

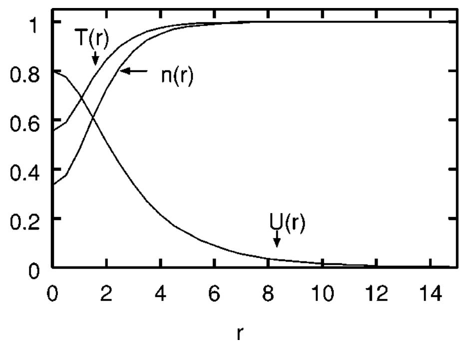

FIG. 3. Normalized em field U, plasma temperature T_0, and density n_0 versus r for $T_\infty = 1$.

sponding ground state solution is stable for $0 < \lambda < \lambda_c$. Notice that the power of the solitary beam always exceeds a certain critical value $N > N_c \approx 7.8$. We also know that N must be bounded from above ($N \leqslant N_m \approx 10.5$).

For arbitrary temperatures, the explicit form of $\Psi = \Psi(|A|^2)$ cannot be found. However, due to its saturating character, solutions with properties similar to the small temperature case (which can be explicitly solved) might be expected. Using relations (8) and (11), we numerically find a stationary solution of Eq. (13) for arbitrary T_∞. For convenience we use following representation of vector potential $A/A_c = U(r)\exp(i\lambda z)$, where $A_c = (G_\infty^2 - 1)^{1/2}$. Although the maximum value of U is still restricted by the condition $0 < U_m \leqslant 1$, the amplitude of the vector potential A_m can reach a considerable value. For ultrarelativistic temperatures, $T_\infty \gg 1$, we have $A_c = \sqrt{15 T_\infty} \gg 1$, and since $0 < A_m \leqslant A_c$ a soliton solution with ultrarelativistic strength of the em field is possible. Here we present the results of simulations for $T_\infty = 1$ (i.e., $T_\infty \approx 0.5$ MeV). The solution exists provided $0 < \lambda < \lambda_c$ (≈ 0.22). The profiles of the field $U(r)$, the plasma density $n_0(r)$, and the temperature $T_0(r)$ are exhibited in Fig. 3 for $\lambda = 0.1$. One can see that in the region of field localization the plasma temperature and density are reduced. Similar plots could be obtained for all allowed values of λ. When $\lambda \to \lambda_c$, plasma cavitation takes place, i.e., at $r = 0$ the plasma density and temperature tend down to zero. The appearance of zero temperature is not surprising since the corresponding region is the "plasma vacuum;" all particles have gone away. The dependence of N on λ is presented in Fig. 2. One can see that the curve $N = N(\lambda)$ has a positive slope and the corresponding solitary solutions are stable against small perturbations.

The detailed dynamics of the arbitrary field distribution must be studied by direct simulations of Eq. (13). Our simulations show that this equation, with the nonlinearity particular to the problem at hand, reproduces the general expected behavior of the NSE with saturating nonlinearities [7]. For all such systems the beam monotonically diffracts if the beam power is below a critical value ($N < N_c$), and it is trapped if $N_m > N > N_c$. In the latter case, the beam parameters oscillate near the equilibrium, ground state values. These oscillations monotonically decrease with increasing z due to the appearance of the radiation spectrum. For larger z, the oscillations are damped out, and the formation of the soliton in its ground state takes place. If the initial profile of the beam is close to the equilibrium one, the beam quickly reaches the ground state equilibrium, and propagates for a long distance without distortion of its shape. For $N > N_m$, the multistream motion of the plasma prevents the system from settling in a steady state.

In conclusion, applying a reductive perturbation technique, the system of relativistic Maxwell-fluid equations was reduced to a 2D nonlinear Schrödinger equation with a saturating nonlinearity. We found that if the strength of the em field amplitude is below the wave breaking limit, the beam can enter the self-trapped regime, resulting in the formation of stable, self-guided 2D solitonic structures. The beam trapping owes its origin to the thermal pressure (which opposes the ponderomotive pressure). In the region of beam trapping, the plasma density as well as its temperature is reduced and under certain conditions these parameters can be reduced considerably (i.e. plasma cavitation takes place).

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[1] P.A. Sturrock, Astrophys. J. **164**, 529 (1971); M.A. Ruderman and P.G. Sutherland, *ibid.* **196**, 51 (1995).
[2] M.C. Begelman, R.D. Blandford, and M.J. Rees, Rev. Mod. Phys. **56**, 255 (1984).
[3] M. L. Burns, in *Positron-Electron Pairs in Astrophysics*, edited by M. L. Burns (AIP, New York, 1983).
[4] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).
[5] A.C.L. Chian and C.F. Kennel, Astrophys. Space Sci. **97**, 9 (1983); R.T. Gangadhara *et al.*, Mon. Not. R. Astron. Soc. **262**, 151 (1993).
[6] T. Tajima and T. Taniuti, Phys. Rev. A **42**, 3587 (1990).
[7] V.I. Berezhiani and S.M. Mahajan, Phys. Rev. Lett. **73**, 1110 (1994); Phys. Rev. E **52**, 1968 (1995); S.M. Mahajan *et al.* Phys. Plasmas **5**, 3264 (1998).
[8] F.T. Gratton *et al.* Phys. Rev. E **55**, 3381 (1997).
[9] V.I. Berezhiani *et al.*, Phys. Rev. E **65**, 046415 (2002).
[10] G.Z. Sun *et al.*, Phys. Fluids **30**, 526 (1987).
[11] L.N. Tsintsadze, Phys. Scr. **50**, 413 (1994); S. Kartal *et al.*, Phys. Rev. E **53**, 4225 (1996).
[12] N.G. Vakhitov and A.A. Kolokolov, Izv. Vuzov Radiofiz. **16**, 1020 (1973) [Sov. Radiophys. **9**, 262 (1973)].